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THEORETICAL ANALYSIS OF SCATTERING OF A LASER BEAM FROM ROUGH S--ETC(U)

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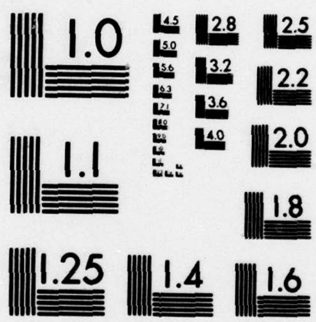

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The accomplishments of the work performed under the support of the grant are summarized. The abstracts of all twelve the publications are given in Appendix A. An important part of the effort in the speckle statistics deals with the computation of the probability density function of light received by a finite size detector. This work is included as Appendix B.

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Final Report

THEORETICAL ANALYSIS OF SCATTERING OF A LASER BEAM FROM ROUGH SURFACES

C. C. Sung

July 1979

U. S. Army Research Office

DAAG29-77-G-0051

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Table of Contents

Foreword

This is the final report of ARO Grant DAAG29-77-G-0051. From the beginning of 1977 to September 1978, the principal investigator was supported 20% by the grant in the academic year and 100% in the summer.

On the basics of the work performed for the Grant the following personnel received degrees:

J. A. Holzer, Ph.D.
W. B. Fowler, Ph.D.
W. D. Eberhardt, M.Sc.

Most of the work reported here was performed in 1979 and 1978. The principal investigator was on his sabbatical from October 1978 to June 1979. During this period he was not supported by the Grant and no effort was made for the task except writing up the results for publication. The report is written in June 1979, when he was away from the Oak Ridge National Laboratory where he held a summer appointment.

In Section 1 the accomplishment of the works performed under the support of the grant is summarized in three categories. The first two categories are directly related to the original proposal and are discussed in details in Section 2A and 2B. The last part contains a summary of the research in some other area. The abstract of all the publications is given in Appendix A. An important part of the effort in the speckle statistics deals with the computation of the probability density function of light received by a finite size detector. This work is included as Appendix B to be submitted for publication later.

Section 1 - Outline of the work performed

The following is a list of publications of the principal investigator arranged in chronical order after the grant was started in the beginning of 1977.

A 1. "Scattering of Electromagnetic Waves from a Rough Surface II" with Holzer, J. of Appl. Phys. 49, 1002 (1978).

A 2. "Scattering of an Electromagnetic Wave from a Very Rough Semi-Infinite Dielectric Plane (Exact Treatment of the Boundary Condition)" J. of Appl. Phys. 49, 994 (1978) with W. D. Eberhardt.

3. "Magnetic Phase Transitions in the Dicke Model," C. Bowden, J. of Phys. A 11, 151 (1978).

4. "Distorted Wave Born Approximation for the Rate Constant of V-V Energy Transfer," with Stettler and Witriol, J. of Chem. Phys. 69, 3112 (1978).

A 5. "Explanation of the Experimental Results of Light Back Scattered from a Rough Surface," with W. D. Eberhardt, J. of Opt. Soc. (AM) 68, 323 (1978).

C 6. "Radiative Energy Transfer through Fog" Appl. Optics 17, 1997 (1978).

A 7. "Scattering of Light from Rayleigh Waves" Appl. Phys. Lett. 32, 399 (1978).

C 8. "Cooperative Bahavior among Three Level System I: Transient Effects of Coherent Optical Pumping," Phys. Rev. 18 A 1558 (1978) with C. Bowden.

B 9. "Probability Density Function of Scattered Light from a Moderately Rough Surface," Appl. Opt. 18, 312 (1979 with Stettler and Holzer.

C 10. "Scattering of Light from a Dielectric Body of Irregular Shapes," J. of Opt. Soc. 69, 756 (1979) with B. Fowler.

C 11. "Phase Transition in the Three-Level Dicke Model", J. of Phys. A (1979) with C. Bowden.

B 12. "Statistical Properties of Reflected Light from a Moderately Rough Surface Observed through Finite-Size Aperature", this work is included in the final report as Appendix B.

All the papers are published in refereed journals. Papers delivered in meetings and conferences are not included.

Publication 3 and 4 are the results of much earlier work and will not be discussed. We divide all other publications in three categories. Category A and B deal with light reflection from a rough surface and speckle statistics that are the main topics which were proposed to be studied in the proposal.

In publication A we show the limitation of the conventional approach of the scattering of light from a rough surface by using the vectorial Kirchhoff integral based on Beckmann and Spizzichino.[1] It is clear that this approach is good for a good conductor and poor for a dielectric body. A realistic surface model is constructed to explain some experimental data and the importance of the boundary condition is demonstrated.

Since the speckle statistics of light reflected from a rough surface has been discussed exclusively for a very rough surface, we compute the probability density function P(I) for a moderately smooth surface both for a point end finite site detector. We show how P(I) can be expressed as a function of the surface parameters.

During 1977 and 1978, we also worked on the laser propagation in various mediums as a part of the Ph.D. dissertation of Mr. Fowler. There was also active collaboration with Dr. C. Bowden on quantum electronics. A brief discussion of these works and the various contributions are given in the section.

Section 2

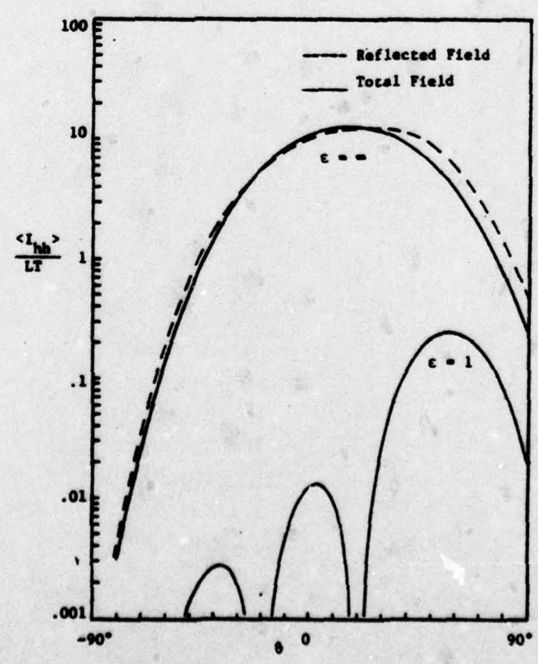
A. Light Scattering from a Rough Surface

Most works in the analysis of experimental data of light scattering of a very rough surface use Beckmann and Spizzichino's work[1] as standard reference. In much earlier works,[2-4] we have shown that the so-called satisfactory results based on Ref. 1 is actually a combination of many approximations, both numerical and analytical involving the treatment of random variables.

In Ref. 5, this issue was carefully studied. It is shown that by using the approximate boundary conditions to relate the scattered field amplitude E_{sc} and the incident field amplitude E_{in}

$$E_{sc} = (1 + r) E_{in} \tag{1}$$

where the sign is chosen for different polarization is a very poor approximation for a dielectric body. This is illustrated in the following figure.



Here the results for very large dielectric constant $\hat{\epsilon}$ are almost independent of the boundary conditions, whereas for $\hat{\epsilon} = 0$, Eq. (1) predicts a reflection of finite magnitude although it is clear that no light should be reflected from a non-existent boundary. It should be emphasized that this conclusion is reached without using any questionable approximation in the treatment of random variables and numerical integration.

The vectorial Kirchhoff integral used in Ref. 1 also fails to predict depolarization of back-scattered light from a rough surface, contradicting the well-known experimental results.[6] In Ref. 7 and 8, a realistic surface model is constructed, so that the surface becomes a combination of independent cells which are described by a quadratic equation. In other words, this surface generalizes the tangent plane approximation to include the curvature correction. When a calculation including exact treatment of the boundary conditions is completed,[7,8] we find that the results not only predict depolarization but also have correct angular distribution. In the following figure, the dashed line is roughly the experimental result, and the solid lines are from the calculation. Notice that the magnitude of the depolarized component is not sensitive to the surface roughness in contrast to the co-polarized component. This result is also in good agreement with the observation.[6] We are not aware of any work in the literature based on the first principle calculation that can derive these results.

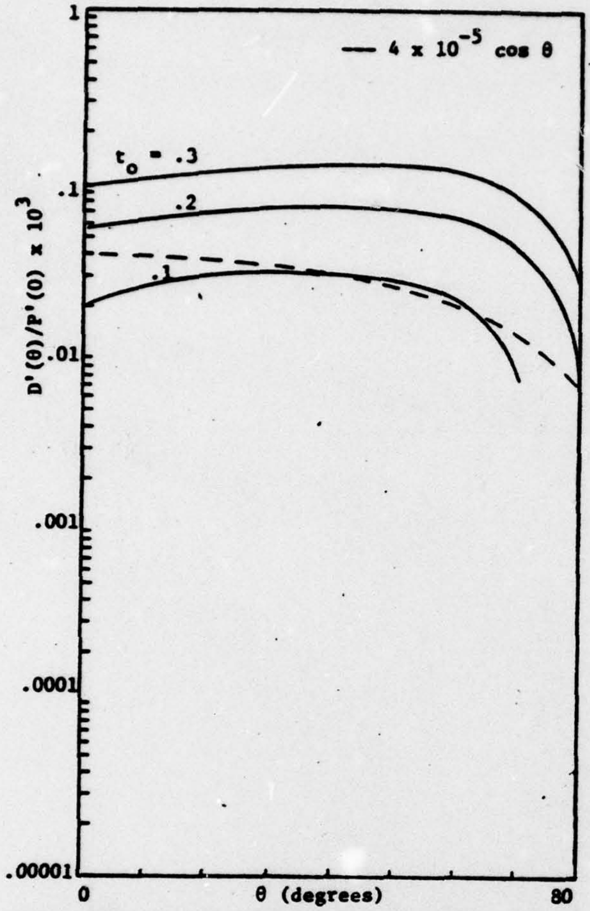


FIG. 3. Angular dependence of the backscattered depolarized component for different incident angles θ and t_o. $h = 0.1$, $\epsilon = 2.5$.

The only undesirable feature is that only small depolarization is predicted whereas the experimental value is much larger. It is possible that this deficiency is caused by neglecting multiple scattering in the calculation.

The importance of the boundary conditions in any calculation of the scattering of light is again demonstrated in Ref. 9 , where the laser light reflection from a corrugated surface generated by acoustic waves propagating in a dielectric medium is computed. Experimental data and previous works on this subject are reviewed by Lean.[10] We show that previous calculation based on Ref.1 actually never predict even the shift of the Brewster angle for diffracted wave of different orders. This shift has been measured experimentally but unfortunate algebraic errors in the earlier work[10] give a wrong impression, that the calculation based on Ref. 1 is satisfactory. Our calculation satisfies the boundary conditions exactly and should be useful for future experimental work in this area.

We should mention many works[11-12] in the recent literature on the light scattering from a moderately rough surface, when the perturbation technique is used to satisfy the boundary conditions. Our works, as a matter of fact, follow their technique closely, with the important difference that we deal with very rough surfaces where considerable modifications are required as we did throughout these publications.

II B. Speckle Statistics

The reflected intensity from a rough surface as discussed in II A is related to the surface parameters. More information on the characteristics of the surface can be obtained by measuring the probability density function P(I), which is determined by scanning across the illuminated rough surface. This subject has been reviewed in the literature by Goodman[13] and Dainty[14] who provided a complete description of the background material. There are also a great number of experimental activities.[13-14]

In both the earlier works and later ones, P(I) is computed by assuming Gaussian statistical properties of the scattered amplitude; real and imaginary part of the amplitude from a circular Gaussian variable. This approach is similar to those in the random signed analysis.[15] The coherent component is considered as the signal and the incoherent component the noise, and is very successful for light reflected from a very rough surface, since the phase scattering amplitude should vary between zero and 2π uniformly. The calculation of P(I) for this case seems to be complete.

In the past year, we concentrated on P(I) for reflected light from a moderately rough surface $\sigma \leq \lambda$ when σ is the rms of the roughness and λ is the wavelength. In this case the problem is somewhat simplified since the amplitude can be expanded in σ/λ. On the other hand, the statistical properties about the amplitude assumed in the previous works are not valid and P(I) should be expressed in terms of the parameters of the surface characteristics. This is what we have

accomplished in Refs. 16 and 17. In the first paper: P(I) for a point detector is considered i.e. the size of the detector is less than that of one speckle. P(I) is derived without using any specific assumption about the surface properties except the illuminated area must be much greater than the size of the correlation cell defined in the surface model. This assumption is sufficient for us to invoke the Central Limit theorem to obtain P(I) in terms of the first two moments of the surface variables Z(x). The final expression for P(I) contains three independent parameters related to the rough surface. We have used this expression to fit the data by U. S. Missile Command, Huntsville, Alabama with improved accuracy.

In Ref. 17, which is included as Appendix B, we generalize the work in Ref. 16 for detector of finite size, where several speckles may appear and one has to consider the fluctuation of many speckles. Again this problem differs from our case to the very rough surface case in that we have only one real random variable Z(x) instead of the real and imaginary component of the amplitude. For a complete general case of arbitrary scattering geometry, the numerical work is too formidable, as indicated in Ref. 17. We work out for a case of practical interest and the numerical results are expressed in terms of relevant parameters, average number N of speckles in the detector, $(\sigma/\lambda)^2$, and the ratio of correlation length to the size of the illuminated area. It is shown that for $N \gg 1$, P(I) is not remarkably different from that of a very rough surface, if the parameters are properly identified. For $N \geq 1$ a careful numerical solution is necessary for accurate results.

II C. Miscellaneous

Refs.18-21 the product of other research activities beside the grant, although they are all related in a broad sense. In Ref. 18, the radiative transfer equation is solved numerically for a medium consisting of aerosols for propagation of a coherent light source of arbitrary intensity distribution. The contribution is mainly numerical, but it should be useful for practical simulation, and important to the application of laser radar technology,[22] since no limitation is imposed on the form of the intensity distribution in our calculation. The loss of coherence and contrast as a function of the propagation distance through the medium is clearly demonstrated in the numerical calculation.

Continuing the effort in Ref. 18, we later address to the scattering of light from a dielectic body of arbitrary size and shpe. This problem is of interest both to laser radar technology[22] where the contribution of anisotropic particles is a subject of concern and applied physics where the Rayleigh's Hypothesis[23] has always been a subject of study. In Ref. 19, we compute the diffracted intensity of an imperfect sphere and compare with the known exact solution[24] to show the limitation of Rayleigh's Hypothesis. We also obtain numerical results consistent with the experimental results in the light scattering of an anisotropic medium.[25] It is confirmed that although the previous empirical[25] treatments are simple and efficient, in some special cases, their results, however, should be used cautiously.

In the later part of 1978, the interests of the principal investigator was shifted toward quantum electronics. Refs.20 and 21

specifically are addressed to the three-level molecules, in contrast to the two-level molecules used in the literature.[26] Our model of course is more realistic. It is found in Ref. 20, however, that the characteristics of the phase transition in the Dicke model is not changed very much except there exist two second order phase transitions instead of one. One important result derived in Ref. 20 is that the transition temperature should be given in rms terms of atoms inside a volume λ^3 instead of the V, the quantization volume which is arbitrary in the Dicke model.

In Ref. 21, we also study the cooperative behavior in the three-level molecules. This model makes it possible to compute effect of coherent pumping. The results seem to explain some of the well-known discrepancy between the two-level model and experimental data.[26-27]

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17. "Statistical Properties of Reflected Light from a Moderately Rough Surface Observed through Finite-Size Aperature," this work is included in the final report as Appendix B.

18. "Radiative Energy Transfer through Fog" Appl. Opt. 17, 1997 (1978).

19. "Scattering of Light from a Dielectric Body of Irregular Shapes," J. of Opt. Soc. 69, 756 (1979) with B. Fowler.

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APPENDIX A

Reprinted from Journal of APPLIED PHYSICS Vol. 49, No. 3, Pt. I, March 1978 A I

Scattering of electromagnetic waves from a rough surface. II

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C. C. Sung

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(Received 23 May 1977; accepted for publication 21 July 1977)

The mean irradiance reflected from a randomly rough dielectric surface is calculated using the vector Kirchoff integral. The tangent-plane approximation is used to relate the fields on the boundary to the incident fields through the Fresnel reflection coefficients. The averages involving the random surface variables are performed exactly using a spectral representation for the scattering function. The special case of a corrugated surface is treated numerically to gain insight into the more general problem. Our results are compared with previous solutions. An empirical formula is presented for practical applications. Finally, a general expression for the depolarization matrix is derived, retaining terms up to second order in the surface slopes. Our solution is compared with available theories.

PACS numbers: 42.10.Hc, 41.10.Hv, 84.40.Ed

Scattering of an electromagnetic wave from a very rough semi-infinite dielectric plane (exact treatment of the boundary conditions)[a]

J. of Appl. Phys.

C. C. Sung and W. D. Eberhardt

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(Received 19 May 1977; accepted for publication 21 July 1977)

49, 494 (1978)

The reflected and refractive fields are calculated for a very rough surface which is assumed to be a set of tangent planes with small corrections due to the local finite radii of curvature. A perturbation technique similar to those used in the moderately rough surface calculations is developed to treat the solution for the tangent planes as the unperturbed solution with the curvature correction as the perturbation. Our results are expressed in terms of the slope distribution function of the tangent planes and the average radius of curvature. Numerical results for scattering of both polarizations are obtained. The depolarization calculation is shown to be consistent with the experimental data. Further, the relationship between the surface model and the conventional Gaussian model is discussed, and a comparison with previous works is given.

PACS numbers: 42.10.Hc, 41.10.Hv, 84.40.Ed

JOSA 68, 323 (1978)

Explanation of the experimental results of light backscattered from a very rough surface*

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(Received 3 October 1977)

We have calculated reflected and refracted electromagnetic waves from a rough surface with the boundary conditions exactly satisfied. The surface model consists of three parameters: the dimension of the independent cells, h; the parameter in the slope distribution function of the tangent plane t_0; and the average radius of curvature of the surface, R. Within the reasonable range of these parameters, we have calculated the polarized component P and the depolarized component D measured by Renau *et al.* and have found the following features of the scattering of the reflected wave from a rough surface that have not been previously explained theoretically. (i) P and D have very different angular dependences. (ii) Whereas P is a sensitive function of t_0, D is almost independent of t_0. (iii) D does not vanish at all angles. A qualitative comparison between the data and the calculation is given.

A2

Scattering of light from Raleigh waves

Appl. Phys. Lett 32, 399 (1978)

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(Received 15 August 1977; accepted for publication 1 February 1978)

The diffracted intensity of light scattered from surface (Rayleigh) waves is calculated by satisfying all boundary conditions to first order in the surface displacements. Comparison with existing experimental data and previous theoretical results is made, and several discrepancies are indicated. More importantly, the relative positions of the Brewster angles for the various diffracted orders appear to the reversed, experimentally, from those predicted in the present treatment.

PACS numbers: 43.35.Sx, 78.20.Hp, 43.35.Pt

Reprinted from **APPLIED OPTICS**, Vol. 18, page 312, February 1, 1979

Probability density function of the intensity of light reflected from a moderately rough surface in the far field

J. A. Holzer, J. D. Stettler, and C. C. Sung

A general expression for the probability density function $P(I)$ of the intensity of light reflected from a moderately rough surface in the far field is derived by the cumulant expansion of the random variable. It is shown that $P(I)$ satisfies the modified Rician statistics for some general surface models in the limit of large L/T, where L is the dimension of the illuminating area, and T is the correlation length.

JOSA 69, 756 (1979)

Scattering of an electromagnetic wave from dielectric bodies of irregular shape

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(Received 4 May 1978)

Cross sections for an electromagnetic wave scattered from dielectric bodies of irregular shape have been obtained. The boundary conditions are reduced to a set of linear algebraic equations whose variables are the expansion coefficients of the electric and magnetic fields. Numerical results are compared with the exact solution for oblate spheroids and good agreement is obtained despite the Rayleigh hypothesis implicitly used in the formulation. The procedures used by Chylek, Grams, and Pinnick to fit the experimental data are also examined and discussed.

A3

Appl. Opt. 17, 1797 (1978)

Radiative transfer in two dimensions through fog

B. W. Fowler and C. C. Sung

The 3-D time-independent unpolarized radiative transfer equation is developed into a form amenable to numerical solution by expansion of the total intensity in orders of scattering. A 2-D numerical algorithm is formulated using an eight-direction quadrature approximation to the source function and an optimized path-by-path intensity integration scheme. Computer code generated from the algorithm is used to examine the multiple scattering effects at 1.06 μm, 3.0 μm, and 10.6 μm in Deirmendjian C3 fog on a finite uniform intensity profile beam and an infinite Gaussian intensity profile beam. Transmission and backscatter for both beams are examined as well as contrast degradation of the uniform beam and spread of the Gaussian beam.

PHYSICAL REVIEW A VOLUME 18, NUMBER 4 OCTOBER 1978

Cooperative behavior among three-level systems: Transient effects of coherent optical pumping

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(Received 18 May 1978)

The dynamical evolution of an excited-state population in a macroscopic volume of three-level molecules is considered where the population is driven optically between the initial ground state and the excited states by an externally applied (c-number) coherent pump. The intermediate and ground states are taken as nonradiatively coupled whereas the excited-to-intermediate-state transition is coupled to the radiation field which is treated quantum mechanically. It is found that stimulated Raman processes can significantly influence the dynamical evolution of population inversion and macroscopic polarization by producing coherence effects among the populations. We examine the evolution of collective relaxation between the excited and intermediate levels in the time regime of the pump pulse duration. It is shown, using the mean-field approximation, that for uniform pumping and conditions such that the pump Rabi rate ω_R, the characteristic collective radiation time τ_R, and the pump pulse duration τ_p satisfy the inequalities, $\omega_R > 1/\tau_R > \gamma$ and $\tau_p \geq \tau_R$ (γ is a characteristic internal dephasing rate for the molecules), the system is left in a state with classical transverse polarization when the pump pulse terminates. The system can evolve collectively from such a state only as superradiant (classical) evolution. For superfluorescent evolution which requires a state of preparation of complete inversion (zero transverse polarization), it is shown that the pump pulse must be effectively of area π and that $\tau_p < \tau_R$. Our results show that when the former conditions are satisfied the delay time τ_D for collective free pulse evolution is a function of both τ_p and τ_R. It is shown further that the pump pulse shape as well as temporal duration significantly affect the final state of preparation. The results of this work are interpreted in connection with recent reported results of experiments in superfluorescence and superradiance.

A4

J. of Phys. (In press)

Phase Transition in the Multi-Mode Two- and Three-Level Dicke Model (Green's Function Method)

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Abstract. By using the thermodynamical Green's function method, we study the phase transition of a collection of two- as well as three-level atoms interacting with the electromagnetic field and contained within a volume much smaller than the smallest resonance wavelength (Dicke model). We show for the case of three-level atoms, the existence of two critical temperatures where a second order phase transition takes place. The upper critical temperature is determined by the largest of the three coupling constants in the three-level model, whereas the lower critical temperature is a function of the remaining two coupling constants. We also show that for a collection of two- as well as three-level atoms, the critical temperature depends upon the number of atoms contained within λ_r^3 (λ_r is a resonance wavelength) rather than the density as has previously been suggested. Also, using the Green's function method, we show the formal equivalence of the Dicke model in thermodynamic equilibrium with the BCS model of superconductivity.

APPENDIX B

Statistical Properties of Light Reflected from a Moderately Rough Surface Scanned Through Finite Size Aperture

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The probability density function $P(I)$ of the integrated intensity I, reflected from a moderately rough surface, is computed correct to $(2\pi\sigma/\lambda)^2$ where σ is the rms of the rough surface $z(x)$ and λ is the incident wavelength. By using the assumption that $z(x)$ is a Gaussian random variable, we obtain $P(I)$ as a function of the size of the detector aperture, and the dependence of the correlation length of the surface model. Numerical results and a comparison with previous works are given.

I. INTRODUCTION

Statistical properties of speckle patterns have been discussed extensively in recent literature.[1] Several review articles[2-4] provide excellent background material. The calculation of the probability density function P(I) for reflected light from a very rough surface, or transmitted through a very diffusive medium, is based on the justifiable assumption that the field amplitudes obey Gaussian statistics. The result, which is independent of the details of the surface properties, is that P(I) is a Gamma distribution function with the order parameter depending on the size of the detector aperture. Numerical results and other applications are given by Seribot[5], Dainty[6], and Barakat[7].

For a moderately rought surface, P(I) has been calculated[8,9] and experimentally[10] observed for a point detector. In References 8-10, P(I) is calculated by assuming that the field amplitudes obey non-circular Gaussian statistics. In Reference 11, we have transferred the assumption of Gaussianity from the field onto the surface variable z(x). Since the field amplitude and z(x) are proportional for a moderately rough surface where $\sigma/\lambda << 1$, the difference between Reference 11 and References 8-10 is only in the definition of various moments that appear in P(I).

We are not aware of any work on P(I) from a moderately rough surface for finite-size detector aperture. An obvious and simple method is to extend the work by Ohtsubo and Asakura[12] who obtained P(I) for a sum of N independent speckles. Their procedure is similar to the earlier work by Goodman[2] who derived a Gamma distribution for the P(I) of a very rough surface for a finite-size detector. If we follow this method, the derived P(I) however, contains a parameter N, the number of speckles in the finite-size detector. N is proportional to the size of the aperture for larger N. For small N, P(I) is more complicated as demonstrated by Seribot.[5]

The purpose of this paper is to calculate P(I) of a moderately rough surface observed through a finite-size aperture correct to the second order in $(\sigma/\lambda)^2$ by assuming the Gaussianity of z(x). Besides this assumption and other simplification involving scattering geometry, no other assumptions are made and we believe that this work and Reference 5 complete the calculation of P(I) for surfaces of moderate and extreme roughness.

In the second section, we present the general principle how P(I) can be computed from the known statistical properties. In the third section a simpler scattering geometry is considered and solved numerically in the last section.

II. GENERAL

We consider the field amplitude $E(\vec{x},\vec{u})$ scattered from a rough, illuminated surface ($\vec{x} = x\hat{i} + y\hat{j}$) located at a distance d from the detecting plane. ($\vec{u} = u\hat{i} + v\hat{j}$, and all vectors in this paper are two-dimensional.) It is assumed that the incident light is linearly polarized and the depolarization of the scattered light is not important so that $E(\vec{x},\vec{u})$ is a scalar function. Generalization of the calculation to a vectorial $E(\vec{x},\vec{u})$ has been discussed by Goodman.[2] We limit our calculation to the second order in $z/\lambda(\lambda = 2\pi/k)$ and expand

$$E(\vec{x},\vec{u}) = f_0 + f_1 kz + f_2 k^2 z^2 \qquad (1)$$

where f_0, f_1 and f_2 are assumed to be real functions of $\vec{x}$ and $\vec{u}$. The imaginary part of $E(\vec{x},\vec{u})$ will be included later. Equation (1) then is a general form for all scattering geometries from which f_0, f_1 and f_2 are determined. Our problem now is to derive P(I) for a given statistical property of z(x). We assume that $z(\vec{x})$ are Gaussian random variables

whose first two moments are given by

$$< z(x) > = 0 \quad , \quad < z(\vec{x})\, z(\vec{x}') > = \sigma^2 \exp\left[- (\vec{x}-\vec{x}')^2/t^2\right]. \qquad (2)$$

Equation (2) replaces the assumption on the Gaussianity of the $E(\vec{x},\vec{u})$ in the treatment of scattering amplitude of a very rough surface.[2-4] Physically $E(x,u)$ is a macroscopical quantity whereas $z(x)$ are the characteristics of the surface property. The assumptions used in this work and previous work are different in nature. Equation (1), however, connects these two quantities and it is clear why they serve the same purpose in the derivation of P(I). It should be emphasized that because of Eq. (2), P(I) in this work depends only on the surface properties.

The major difference between our problem defined earlier and other works in the literature is that only one real random variable $z(x)$ is present in both the real and imaginary part of $E(\vec{x},\vec{u})$ whereas in the calculation of P(I) from random signals[13,14] and P(I) for light scattered from a very rough surface,[2-3] the real and imaginary parts of the amplitude are assumed to be independent Gaussian random variables. Roughly speaking, P(I) for light reflected from a moderately rough surface is generated by real Gaussian variables $z(x)$ instead of circular complex Gaussian variables.

$z(x)$ is expanded in terms of othogonal functions $\psi_i(x)$ and random variables S_i

$$z(x) = \Sigma\, S_i \psi_i(x) \quad . \qquad (3)$$

It can be easily verified that $\psi_i(x)$ is a solution of the integral equation

$$\int < z(\vec{x})\, z(\vec{x}')> \; \psi_i(x')\, d^2x' = \lambda_i \psi_i(\vec{x}) \qquad (4)$$

where λ_i is the eigenvalue. Since S_i are linear combinations of Gaussian

random variable z_i, then

$$< S_i > = u \quad . \tag{5}$$

From Eq. (2) and

$$< S_i S_j > = S_{ij} \lambda_i \quad , \tag{6}$$

S_i then have the distribution function given by Eqs. (5) - (6), or ,

$$D(S_i) dS_i = dS_i \ \exp \left(- S_i^2 \big/ 2\lambda_i\right) \Big/ \sqrt{2\pi\lambda_i} \quad . \tag{7}$$

The intensity received in the detector I is explicitly given correct to the second order in $k^2\sigma^2$ by

$$I = \int \Big[\Big(f_o(\vec{x},\vec{u}) \ f_o(\vec{x}',\vec{u}) + 2 \ k^2\sigma^2 \ f_o(x,u) \ f_2(x',u') \Big)$$

$$+ \ 2 \ f_o(\vec{x},\vec{u}) \ f_1(\vec{x}',\vec{u}) \ kz(x') + k^2 \ f_1(x,u) z(x) \ f_1(x',u) \ z(x') \Big]$$

$$d^2x \ d^2x' \ d^2u \quad , \tag{8}$$

where k^2z^2 is set to be $k^2\sigma^2$, since $k^2(z^2-\sigma^2) = \delta$ may be neglected. This is justified by the observation that the average of δ and any function $W(kz)$

$$< \delta W(kz) > = \sum_{n=o}^{\infty} < \delta (kz)^n > \big/ n! \ \left. \frac{d^n \ W(kz)}{d(kz)^n} \right|_{kz=0} \tag{9}$$

is of order $(kz)^4$, since $n = 0$ and $n = 1$ in Eq. (9) both vanish.

Consequently δ can be neglected. Equation (8) can be written as $I_t = I + \bar{I}$, where I_t is the total intensity received at $\vec{u}$ and $\bar{I}$ is from the imaginary part of the amplitude similar to Eq. (8). Equation (8) can be written in the form,

$$I = I_0 + I_1 + I_2 \tag{10}$$

where I_i contains z(x) of i^{th} order. $\bar{I}$ is defined as a sum of terms $\bar{I}_0$, $\bar{I}_1$, and $\bar{I}_2$ similar to Eq. (10).

Equation (3) expresses I as a quadratic function of S_i

$$I = \alpha + \sum_i \beta_i S_i + \sum \gamma_{ij} S_i S_j \tag{11}$$

where α , β_i , and γ_{ij} are constants obtained after integration of the product of ψ_i and various functions in Eq. (10).

P(I) is defined by

$$P(I) = \int \frac{dZ}{2\pi} \exp(i\, I_t Z) < \exp(-i\, I_t Z) > \tag{12}$$

where $I_t = I + \bar{I}$, and < > is understood to be

$$< (\quad) > = \prod_i \int dS_i\, D(S_i)\, (\quad) \; . \tag{13}$$

The calculation of P(I) in Eq. (12) can be summarized as follows: first solve for Eq. (4) to obtain ψ_i and S_i. Next determine α , β_i and γ_{ij} in Eq. (11). Finally, perform the integral P(I) defined in Eq. (12). Although Eq. (12) looks much more complicated than

that derived for a very rough surface, the calculation in general can be greatly simplified as shown later. It should be clear, however, that for P(I) for a moderately rough surface, the real and imaginary of the amplitude cannot be separated as independent variables. Because of this reason, γ_{ij} in Eq. (11) are not diagonal.

III. SIMPLE GEOMETRY

In order to demonstrate how P(I) depends on various parameters, a simpler scattering geometry for numerical calculation is considered here. It should be clear that other calculations are sufficiently general for many similar systems.

We consider the light source of the Gaussian distribution,

$$E_0(\vec{x}) = E_0 \exp\left[-(x^2 + y^2)/2T^2\right]\Big/2\pi T^2 \tag{14}$$

reflected from a rough surface as shown in Figure 1. A lens of focal length f is used to simplify the calculation. $E(\vec{x},\vec{u})$ is explicitly given by

$$E(\vec{x},\vec{u}) = E_0(x)\, G_0(\vec{u})\ xf \cdot \exp\left(i(\vec{k}-\vec{n})\cdot\vec{x}\right) \cdot$$
$$\left[1 + i(k_z - n_z)\cdot z - (k_z - n_z)^2\, z^2/2\right] \tag{15}$$

where $G_0(\vec{u}) = \exp\left[\frac{ik}{2f}\left(1 - \frac{d}{f}\right)(u^2 + v^2)\right]$[15]; and $\vec{k}$ and $\vec{n}$ are the propagation and the scattered direction. Notice that $\vec{n} = k\vec{u}/f$ depends on $\vec{u}$.

The intensity I received in the detector is

$$I = \int d^2u \int d^2x d^2x'\ E_0(x)\, E_0(x')\, \exp i(\vec{k} - \vec{n})(\vec{x} - \vec{x}')$$
$$\times \left[1 - (k_z - n_z)^2\sigma^2 + i(k_z - n_z)(z(x) - z(x')) + (k_z-n_z)^2 z(x)z(x')\right] \tag{16}$$

or

$$I = \int d^2u \left(J_o(u,u) + J_2(u,u) \right) = I_o + I_2 \tag{17}$$

where J_o and J_2 are functions of $\vec{u}$ only and J_2 contains the random variables $z(x)z(x')$. The term linear in $z(x)$ vanishes as shown later. In order to compute $P(I)$ in this special case, we find it simpler to follow many previous works and define the mutual intensity function $J_2(\vec{u},\vec{u}')$ and solve the integral equation

$$\int J_2(\vec{u},\vec{u}')\, \phi_i(\vec{u}')\, d^2u' = \lambda_i \phi_i(\vec{u}) \tag{18}$$

where

$$J_2(\vec{u},\vec{u}') = \frac{1}{\lambda^2 f^2} \int (k_z - n_z)^2\, z(\vec{x}) z(\vec{x}')\, \exp\left[i\vec{k}(\vec{x}-\vec{x}') - i(\vec{n}\vec{x} - i\vec{n}',\vec{x}') \right]$$

$$X\ E_o(\vec{x})\, E_o(\vec{x}')\, d^2x d^2x' \tag{19}$$

which can be written as

$$J_2(u,u') = \frac{E_o^{\,2}}{\lambda^2 f^2}\, t^2 \quad \exp\left[-t^2 \left(\vec{k} - \frac{\pi \vec{u}_+}{\lambda f} \right)^2 - T^2 \left(\frac{\pi \vec{u}_-}{\lambda f} \right)^2 \right] \tag{20}$$

where $u_{\pm} = \vec{u} \pm \vec{u}'$ and $t << T$ has been used. Notice that J_2 becomes a product of two functions of u_+ and u_-, respectively. The u_- dependence comes from the distribution of the incident intensity, whereas u_+ dependence contains the surface parameters. This is the expected result for any incident distribution much coarser than t . In the limit of small t or zero correlation length, $< J_2 >$ depends only on u_- , which is the Van-Cittel-Zenike theorem.[16]

The solution ϕ_i from Eq. (18) forms a complete, othonormal set, from which we expand

$$\int (k_z - n_z) z(\vec{x}) \exp\left[i(\vec{k}-\vec{n}) \cdot \vec{x}\right] E(x)\, d^2x = \Sigma\, S_i \phi_i(u). \qquad (21)$$

Similar methods used earlier show that the random variables S_i satisfy

$$< S_i > = 0 \quad , \qquad (22)$$

$$< S_i S_j > = \lambda_i \quad . \qquad (23)$$

Notice that the expansion Eq. (21) is different from Eq. (3), which is used for the purpose of expressing both real and imaginery part of the amplitude in terms of ψ . In our simple geometry, Eq. (21) is sufficient to diagonalize I_2

$$I_2 = \Sigma\, S_i^2 \quad , \qquad (24)$$

Although in principle, we still have to find z(x) as an explicit function in order to express the linear term in Eq. (17) which can be done by using Eq. (21), in our practical solution the linear term is proportional to the imaginary part of $\sum_i S_i \phi_i$ and vanishes since S_i and ϕ_i are real as a result of the approximation in Eq. (20).

P(I) from Eq. (12) is

$$P(I) = \int_{-\infty}^{\infty} \frac{dZ}{2\pi} \cos\left[z(I-I_0) - \Sigma\theta_i/2\right]\prod_i (1 + 4\lambda_i^2 Z^2)^{-\frac{1}{4}} \tag{25}$$

where $\theta_i = \tan^{-1}(2\lambda_i z)$.

We will use the one-dimensional model in the numerical calculation to discuss how P(I) depends on various parameters. Eq. (18) then is written in the form for the normal incident wave,

$$A\,\sigma^2 k^2 c \int_{-1}^{1} \exp(-Sc\,u_+^2 - c\,u_-^2)\,du'\,\phi_i(u) = \lambda_i\phi_i(u) \tag{26}$$

where $S = t/T$ and u is the unit of ℓ , where ℓ is a dimension of the detector. $c = \pi(T\ell/\lambda T)^2$ and c/π^2 may be interpreted as the number of speckles in the detector. A is a constant, and is set to be one, since P(I) depends only on the relative magnitude of J_0 and J_2; and I_0 has been chosen as a parameter in the following discussion.

The effect of $S << 1$ in Eq. (26) in general is to reduce the eigenvalues of λ_i and hence reduce the contrast as easily seen later. We will discuss first the case $S = 0$ in the perturbation calculation. When $c = 0$, there is only one eigenfunction, ϕ_i = const., and one eigenvalue, $2\sigma^2k^2c$. For small c , the kernel can be expanded on $2\sigma^2k^2c\,(1 - cu_-^2)$ and the eigenfunctions can be written as $\phi_i(u)$,

$$\phi_i(u) = A_i + B_iu + C_iu^2 . \tag{27}$$

By substitution of Eq. (27) to Eq. (26), we identify the terms $\propto u^n$ $(n = 0,1,2)$ in both sides, and obtain the equation for eigenvalues λ_i $(i = 0,1,2)$

$$\bar{\lambda}_i(\bar{\lambda}_i - 2/3\ c)\left(\bar{\lambda}_i - (1 - 2/3\ c)\right) = 0 \qquad (28)$$

if we neglect the c^2 term, and $\bar{\lambda}_i \equiv \lambda_i / 2\ c\, k^2\sigma^2$.

For large values of c the kernel becomes a delta function. Many of the eigenvalues are approximately equal and satisfy the same rule

$$\sum_{i=1}^{\infty} \lambda_i = .2\ c\ \sigma^2 k^2 \quad . \qquad (29)$$

In Figure 2, the distribution of the eigenvalues is given. It is interesting to note that there is considerable qualitative differences between the distribution of the eigenvalues in the kernel in Eq. (26) and the sinc function used in Reference 5.

For a set of equal eigenvalues, P(I) can be easily evaluated by[16]

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} dz\, \exp\left[-i\,(I-I_0)z\right] \Big/ \left[1 - i\lambda_0 z\right]^n = \frac{1}{\lambda_0}\left(\frac{I-I_0}{\lambda_0}\right)^{n-1} \exp\frac{\left[-(I-I_0)/\lambda_0\right]}{\Gamma\ (n)} \qquad (30)$$

where n is an arbitrary constant with $R_e n > 0$. The contrast c_t on the basis of Eq. (30) is given by

$$c_t = \left[\bar{I}_0/\lambda\sqrt{n+1} + \sqrt{n+1}\right]^{-1} \quad . \qquad (31)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

P(I) in this work depends on three parameters, $\bar{I}_o$, c , and S in Eqs. (17) and (26). The dependence on $\bar{I}_o$ only shifts the origin of P(I) and changes the value of c_t according to Eq. (31). The dependence of P(I) on c and S is through the distribution of the eigenvalues shown in Figure 2, which then effect the general features of P(I).

Let us examine the c dependence by setting S = 0, I_o = 0. In Figure 3, we plot P(I) for c = 0.1, 1, and 10. c_t of each case are, respectively, 1.24, 0.95, and 0.53. Notice that c_t can be greater than one. In general these curves behave similarly to those for P(I,G) for a very rough surface.[5] Recall that P(I,G) is obtained by solving for similar eigenvalue equations, Eq. (18), except the power in the integral in Eq. (25) is replaced by one, instead of 1/2. It is clear from Figure 3, that for small c , P(I) and P(I,G) are very different at small value of c ; however, P(I) and P(I,G) are very similar, as shown in Figure 4, where c = 20 is used. As a matter of fact, P(I) behaves very much like P(I,G) at a smaller value of c . Eq. (20) has a set of eigenvalues of about the same order of magnitude. We also have fit these curves by using Eq. (23), where I_o and n are determined from the actual curve n = 2.7 and 6.5, respectively, for P(I) and P(I,G). The fact that Eq. (23) fits P(I,G) almost perfectly, has also been demonstrated in Reference 5, where the fit for P(I,G) of smaller value of c was also shown numerically not very good.

Next, we discuss how the P(I) depends on S , which is essentially the reciprocal of the number of correction cells in the target. We should emphasize that the S dependence is very much related to the surface model and the scattering geometry. The appearance of S in Eq. (20) also explains the effect of S on the eigenvalues in Figure 2. First, S does not change the eigenvalue for small c . As c becomes larger, S_i in Eq. (22) is reduced according to the scale in $1/\sqrt{S}$. As a result, the distribution of S_i for finite S is similar to that of S_i for zero S value and smaller c . This feature of the effect of finite value of S (finite number of the correlation cells) is shown in Figure 5, where P(I) is plotted for c = 40 and S = 0, 0.1, and 0.2; c_t = .44, .77, and .97, respectively. A comparison of Figures 5 and 3 shows that P(I) (c = 40, S = 0.2) is rather close to P(I) (for c = 1, S = 0). Notice that for S = 1, there will be only one eigenvalue and γ (u) = exp $(- 2 c\mu^2)$. The result, however, is not meaningful as T > > t , S < < 0 has already been assumed.

The only detailed experiment relevant to our calculation is in Reference 10, where P(I) is only measured for c ≈ 0. P(I) of a point detector is more appropriate for direct comparison, although the qualitative dependence of P(I) on I_0, and S in Reference 10 is in agreement with the calculation. It is important to note that $\bar{I}_0$ may be a complicated function of S , and c^{10} , and $\bar{I}_0$ may dominate the magnitude of c_t in Eq. (24). Thus, the experimental control of $\bar{I}_0$ is crucial for a qualitative comparison with the calculation.

In summary, we have obtained P(I) for a reflected light from a moderately rough surface as a function of the number of correlation cells in the object, and the size of the scanning aperture. The calculation supplements Seribot's work[5] on P(I) for a rough surface.

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FIGURE CAPTIONS

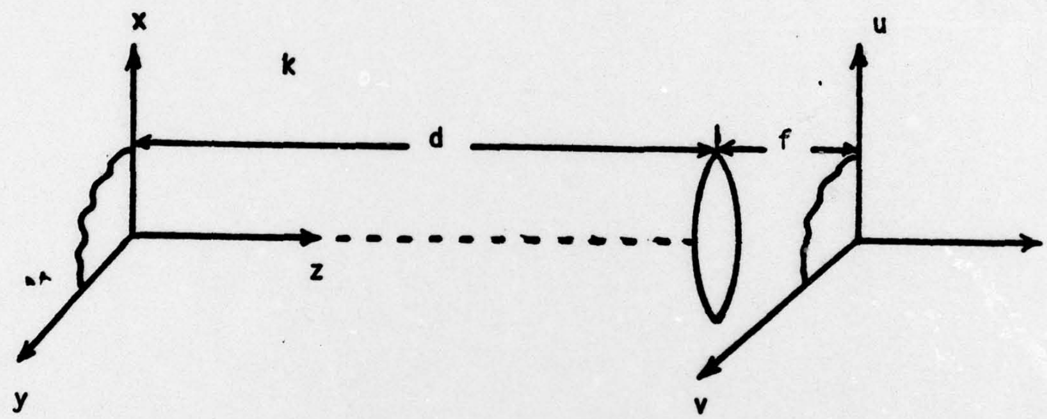
Figure 1. The geometry of the object and observation plane.

Figure 2. Distribution of the eigenvalues for various parameters. Curves A, B, and C are for S = 0, c = 1, 10, and 100, respectively. B' and C' are for S = 0.1, c = 10 and 100, respectively.

Figure 3. P(I) for various c and S = 0. Curves A, B, and C are for c - 10, 1, and 0.1, respectively. Each curve is multiplied by a factor γ , $\gamma_A = 0.2$, $\gamma_B = 1.3$, $\gamma_c = 5.3$

Figure 4. P(I) for different statistics. Solid curves A and B are based non-circular and circular Gaussian statistics. The dashed curve is calculated from Eq. (23).

Figure 5. P(I) for c = 40 and various S . Curves A, B, and C are for S = 0, 0.1, and 0.2, respectively. P(I) is multiplied by 40.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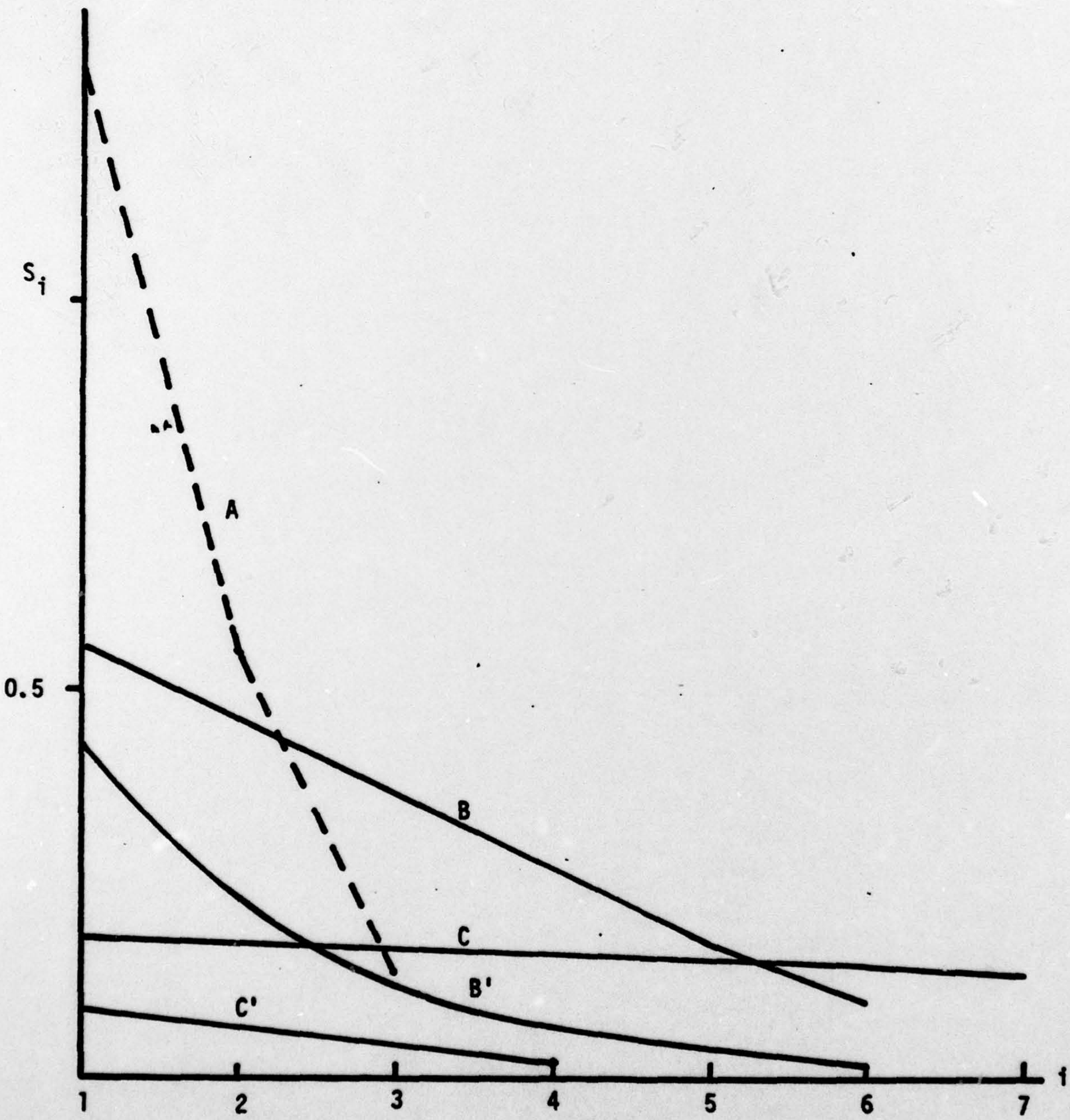

S_i
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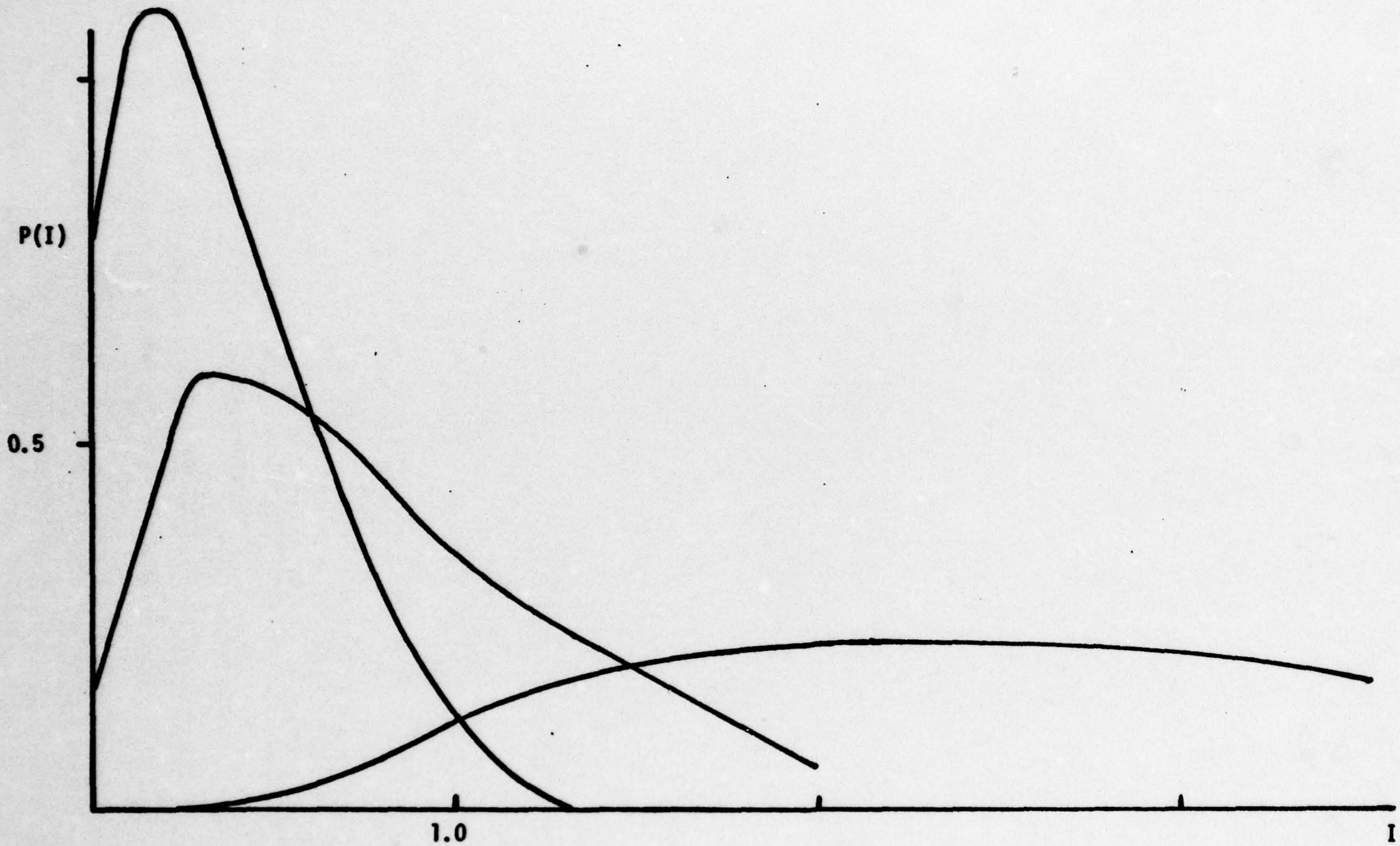
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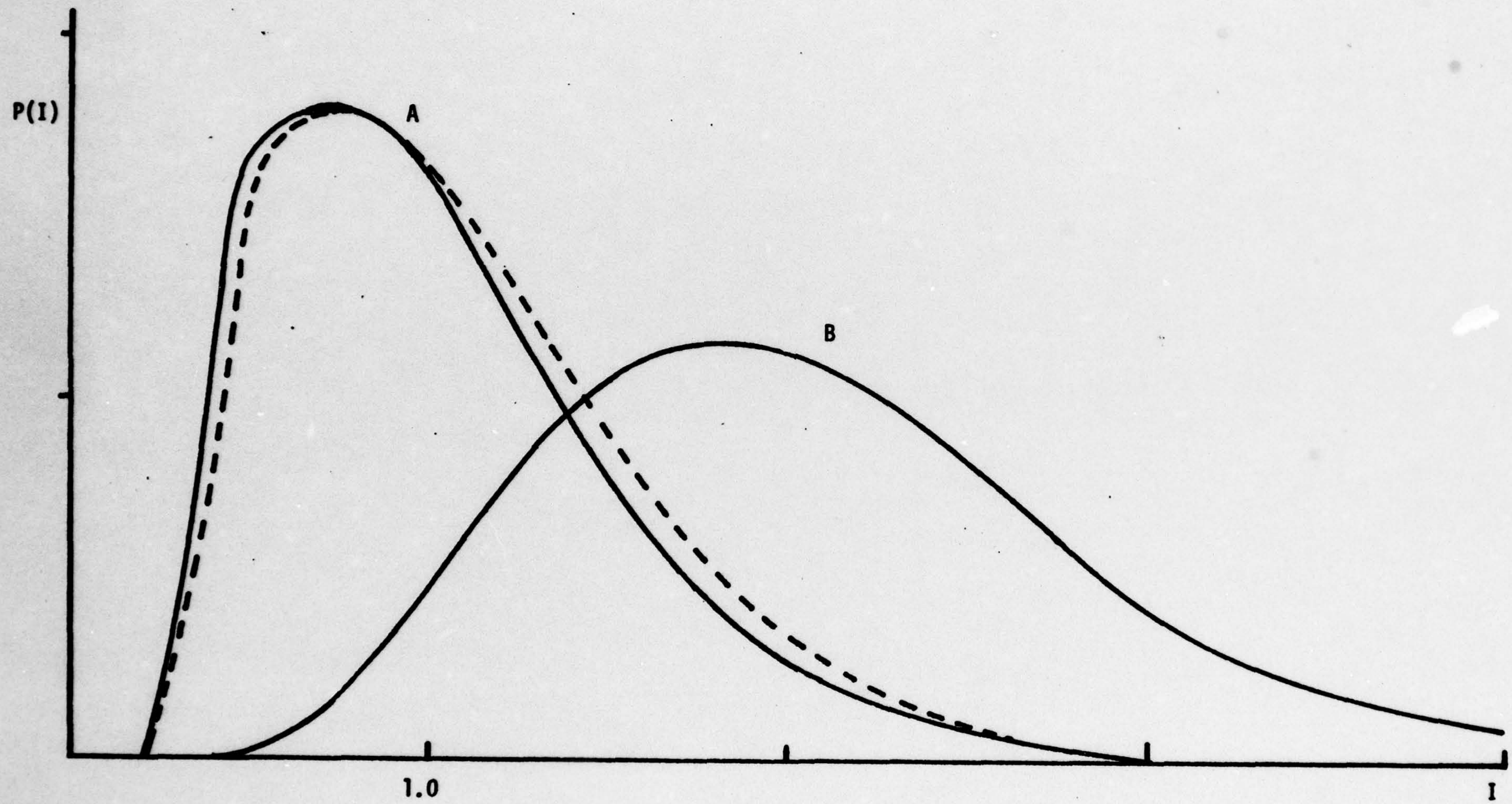
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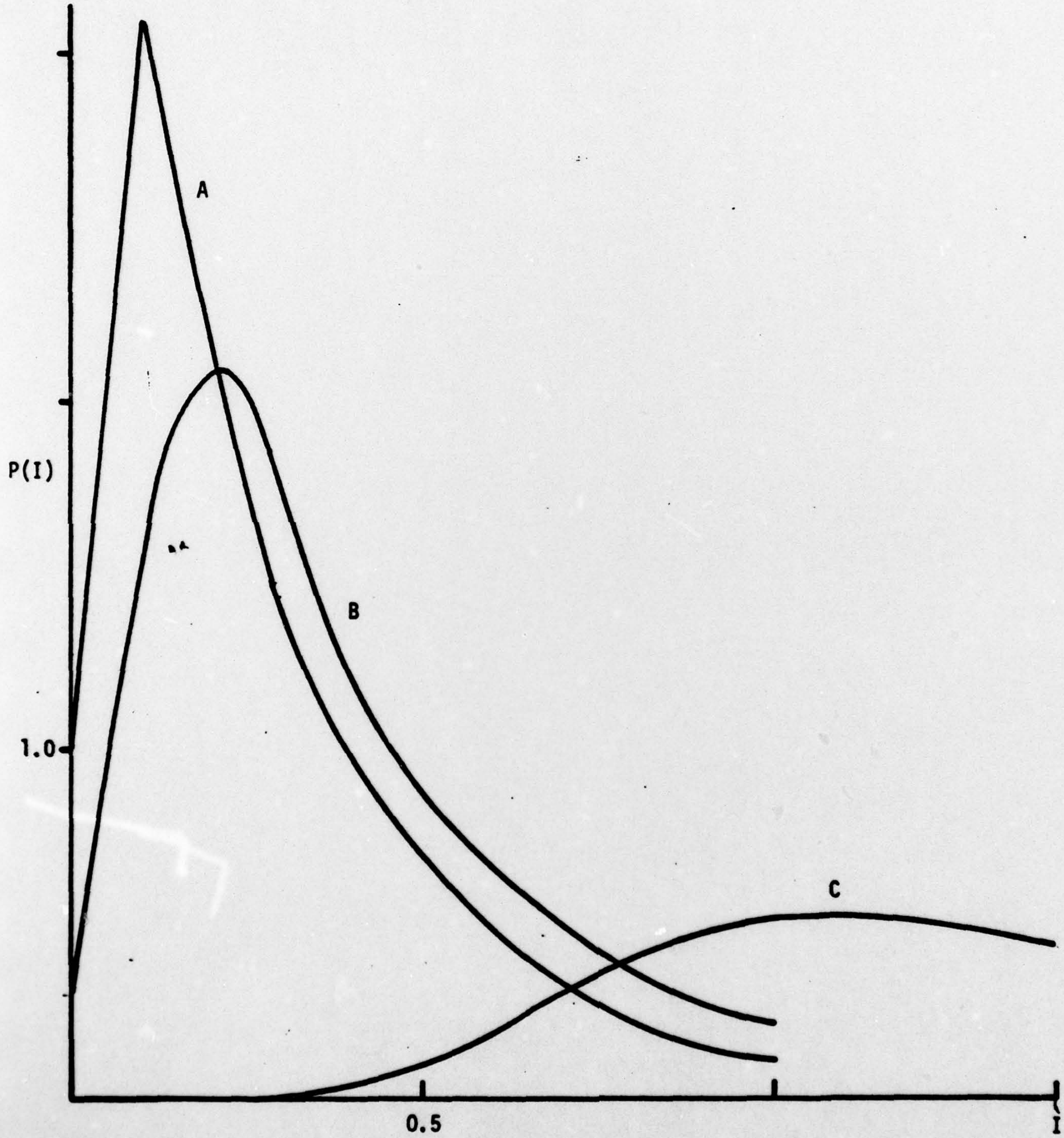

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